Forward Kinematics: (DH representation)

The forward kinematics can be stated as follows: Given the joint variables of the robot, determine the position of the end effector

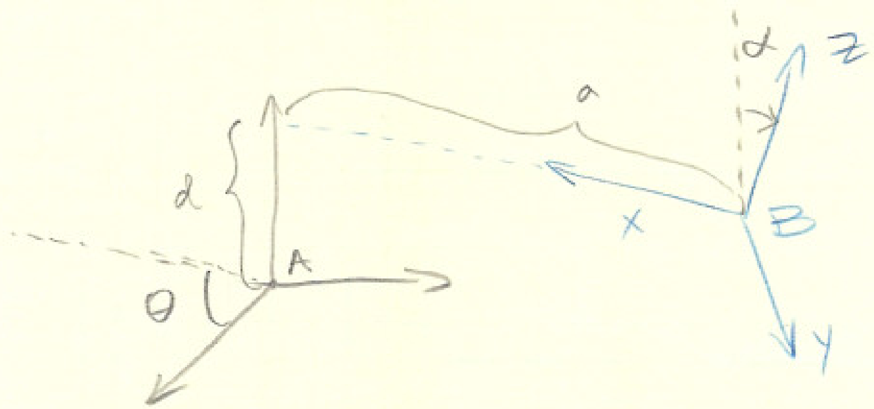
The joint variables are the angles between the links in the case of revolute joints and the link expeasion in the case of priomatic joints

Note: z axis is the direction of the displacement, Rotation happens about the z axis.

So what we want to do with this repersentation is always be able to describe the position and rotation by 4 parameters rather then 6.

This happens by

$$Rot_z(\theta)\,Trans_z(d)\,Trans_x(a)\,Rot_x(\alpha)$$



Note: x in frame B must be pointing at z in frame A

EX

$$Rot_z(\alpha) \rightarrow \underset{\text{fixed}}{Trans_x(a)} \longrightarrow \underset{\text{fixed}}{Rot_y(\beta)}$$

$$\rightarrow \underset{\text{moving}}{Rot_x(\gamma)} \rightarrow \underset{\text{moving}}{Trans_z(b)} \rightarrow \underset{\text{fixed}}{Rot_x(\delta)}$$

SOL first we forget about the translations.

$$R = R_x(\delta) \cdot R_y(\beta) \cdot R_z(\alpha) R_x(\gamma)$$

To find d we first take all the fixed translations and add them together the translation w.r.t. the moving frame will be affected by all the rotations before it.

$$d = \overset{trans_x(a)}{\begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix}} + \overset{trans_z(b)}{\begin{bmatrix} 0 \\ c \\ 0 \end{bmatrix}}$$

$$+ R_y(\beta) R_z(\alpha) R_x(\gamma) \overset{trans_z(b)}{\begin{bmatrix} 0 \\ 0 \\ b \end{bmatrix}}$$

$$\therefore \quad \underbrace{T}_{\text{same as } H.} = \left[\begin{array}{c|c} R & d \\ \hline 0\;0\;0 & 1 \end{array}\right]$$

Therefore again, with regard to the DH reperesentation.

if $\boxed{x_1 \perp z_0}$ and $\boxed{x_1 \text{ intersects } z_0}$

Then 4 unique paramuters, a, d, α, θ such that the homogeneous transformation from frame 0 to frame 1 and is described by.

$$A = Rot_z(\theta) Trans_z(d) Trans_x(a) Rot_x(\alpha)$$

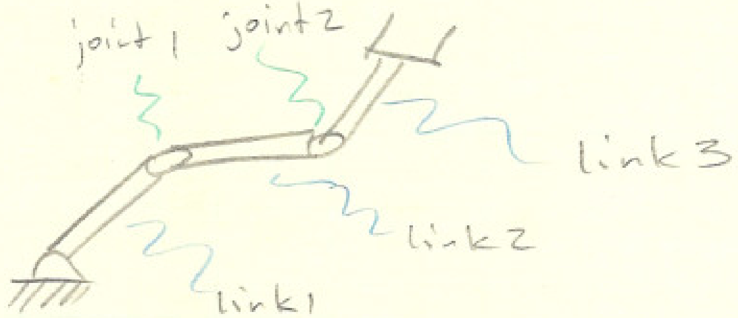
Conditions

Assume we have n degrees of freedom

$\therefore$ n links

link 0 is base

The joints are numbered 1 to n.

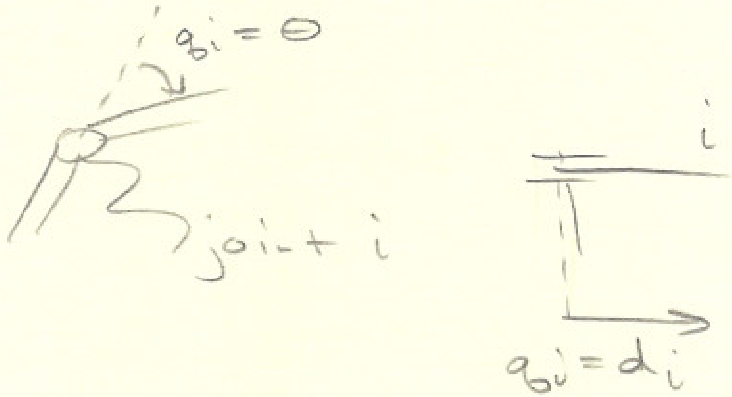


The ith joint is the point is space where links $i-1$ and i are connected.

The ith joint variable is denoted by q_i

q_i : is the angle of rotation in the case of the revolution joint

q_i: is the linear displacement in the case of a prismatic joint.



A coordinate frame is attached to each link

Frame 0 is attached to the base (AKA fixed frame)

Frame i is rigidly attached to link i.

Let Ai be the homogenous transformation from fram i-1 to frame i

$$A_i = A_i(q_i)$$

joint variable for joint i.

The homogenous transformation that transforms a point j in frame i is:

$$^{i}_{j}T = A_{i+1} \cdot A_{i+2} \cdots A_j$$

$$i < j$$

$$ {}^{i}_{i}T = I $$

$$ {}^{i}_{j}T = ({}^{j}_{i}T)^{-1} \quad \text{if} \quad i < j $$

Let the end effector position and orientation be

$$ {}^{0}_{n}P \quad \& \quad {}^{0}_{n}R $$

$$ {}^{0}_{n}T = \left[\begin{array}{c|c} {}^{0}_{n}R & {}^{0}_{n}P \\ \hline 0\ 0\ 0 & 1 \end{array}\right] $$

${}^{0}_{n}P$: coordinate of the origin of frame n w.r.t. frame 0

${}^{0}_{n}R$: rotation of the frame n w.r.t. frame 0

$$ A_i = \left[\begin{array}{c|c} {}^{i-1}_{i}R & {}^{i-1}_{i}P \\ \hline 0\ 0\ 0 & 1 \end{array}\right] $$

$$ {}^{i}_{j}T = A_{i+1} \cdots A_j $$

$$ {}^{i}_{j}R = {}^{i}_{i+1}R \cdots {}^{j-1}_{j}R $$

$$ {}^{i}_{j}P = {}^{i}_{j-1}P + {}^{i}_{j-1}R\,{}^{j-1}_{j}P $$